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X-ray Studies of Polycrystalline Metals Deformed by Rolling. III. The Physical Interpretation of the Experimental Results

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The experimental results on the deformation of polycrystalline metals, presented in earlier papers, are discussed in terms of dislocation mechanisms. A 'foam' structure of particles is produced by the deformation. The particles are regions of low dislocation density separated by boundaries of high dislocation density. It is suggested that the particles are the regions between the slip bands and that misorientations occur at the slip bands. Satisfactory agreement is obtained between the observed particle sizes and minimum slip-band spacings. The dislocations producing curvature are distributed over considerable volumes of material around the slip bands. The formation of particles is thought to be a direct consequence of the production and locking of slip bands during the deformation. The boundaries are regarded as obstacles to slip and the strength of a metal is related to the stress concentration necessary to transmit slip from one particle to the next. Satisfactory agreement is found between theoretical values of strength calculated on this basis, using a method given by Frank, and the experimental values.

The mechanisms of recovery and recrystallization are discussed in the light of the experimental evidence.

The experimental results are found to be in agreement with those of other workers in this field, with the exception of those of Wood; it is shown that a critical examination of the latter data may resolve some of the discrepancies. Some suggestions for further researches in this field are made.

1. Introduction

In earlier publications (Hirsch & Kellar, 1952; Hirsch, 1952*a*, *b*; Gay & Kelly, 1953*a*, *b*) the results of experiments on the structure of specimens of soft and hard metals deformed by rolling have been described. In this paper these results are examined with a view to the discovery of common features in the behaviour of deformed polycrystalline metals, and these common features are described in terms of physical processes occurring within the metals. The paper is divided into three parts; Part I contains a brief recapitulation of the results of the experiments, whilst Part II and Part III contain an interpretation of these results and a discussion of their relationship to the experiments of other workers.

PART I

2. Deformation by rolling at room temperature

The experiments have shown that after a small deformation of any metal, the original grains break up into a number of smaller particles which are slightly misorientated relative to one another. The size of these particles decreases with increasing deformation, whilst at the same time there is an increase in the total angular spread of misorientated material within the grains. After sufficient deformation, the particle size reaches a lower limit. The particles within a deformed grain are linked by severely distorted boundary regions. The particles themselves may be distorted, the amount of distortion increasing with the degree of deformation; if it is assumed that the distortion of the particles is due to elastic stresses, it may be calculated that these stresses are of the order of the yield stress for the material.

The metals may be conveniently classified into two groups: (1) The soft metals (Sn, Cd, Zn and Pb) which recrystallize spontaneously after a certain deformation at room temperature. The recrystallized grain size decreases with increasing deformation. (2) The hard metals (Al, Cu, Fe and Ni) which do not recrystallize spontaneously at room temperature.

Thus the textural changes may be illustrated by schematic diagrams (Fig. 1); for detailed results, reference may be made to the publications mentioned above. It appears that all metals behave similarly, the difference between them being of degree rather than of kind. The essential feature of deformation is the production of localized regions of great distortion separating relatively perfect regions.

Certain conclusions may be drawn about the behaviour of metals subjected to rolling:

(*a*) The limiting particle size (t_l) is independent of the initial grain size over the range investigated ($\sim 20-100\mu$ for Al).

(*b*) t_l is, on the whole, larger the softer the metal.

(*c*) There is more heavily distorted material in the harder metals.

(*d*) At least for aluminium, t_l depends on the purity of the metal; it decreases with increasing amounts of impurities.

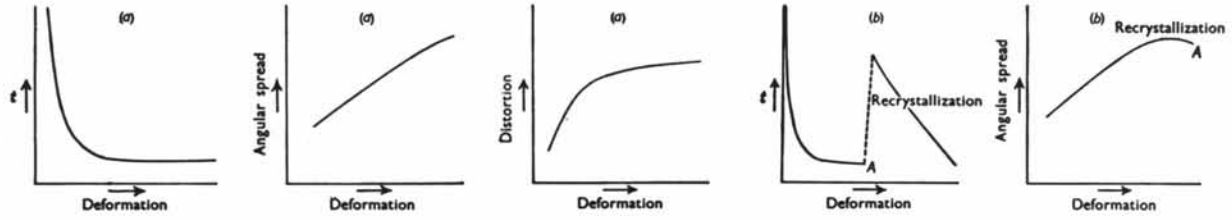

Fig. 1. Schematic diagram of textural changes as a function of deformation for (*a*) hard and (*b*) soft metals.

PART II

3. Interpretation of experimental results by dislocation mechanisms

The existence of relatively perfect regions linked by distorted regions within a cold-worked metal indicates that the distribution of dislocations is not uniform. The particles contain few excess dislocations, whilst the density in the boundaries is much greater.

It is of great importance to ascertain the mechanism by which this distribution of dislocations in the cold-worked state is reached. Two processes which can lead to the observed distribution are as follows:

(i) The dislocations produced by the deformation are distributed throughout the metal, and subsequently segregate into localized regions (e.g. polygon walls) to reduce the strain energy; thus the final state is reached by a process of recovery.

(ii) The dislocations accumulate locally under the influence of stress during the deformation, owing to a 'locking' of one or more dislocations.

It has been suggested that the first process is operative in cold-worked aluminium (Heidenreich, 1951; Hirsch, 1952*b*). However, it is found that the particles are distorted after deformation, and that the stresses are of the order of the yield stress of the material at the time at which it was examined. Further, boundary walls formed by this process would not be highly distorted, whereas it is found experimentally that the boundary regions are so distorted; such objections lead to the serious consideration of the latter process.

Dislocations may be locked in kink bands (e.g. Mott, 1951) or on slip bands by the formation of sessile dislocations (Lomer, 1951). Gay & Honeycombe (1951) have examined with an X-ray microbeam the region covering one kink band on a slightly deformed single crystal of aluminium. The Laue spots consist in general of two diffuse streaks, presumably due to the regions of the crystal on either side of the kink band. These streaks contain a number of ill-defined parallel striations (Fig. 2); comparison with spotty microbeam photographs from polycrystalline material leads to the conclusion that the spots should be identified with these finer striations. It follows that the particles are not related to the kink bands which can be observed by ordinary microscopic techniques; while the possibility of kink bands or other deformation bands occurring on a scale too small to be detected by ordinary methods cannot be ruled out (e.g. local curvature), these considerations lead us to suggest *that the particles are the regions between slip bands.* The following evidence supports this hypothesis:

(1) If the deformation is sufficiently great, slip would take place on all available slip planes and a 'foam' structure would be formed in which the mean particle

Fig. 2. Enlarged Laue streak from a microbeam photograph of one kink band on the surface of a lightly deformed single crystal of aluminium. Fine parallel striations may be seen running diagonally from the top left hand corner.

size is equal to the limiting distance between the slip bands. Satisfactory agreement is found (Table 1).

(2) The particles would be expected to be approximately equiaxed; this has been observed experimentally by Heidenreich (1951) and Hirsch (1952*b*).

(3) In order for the particles to be distinguished,

Table 1. *Comparison of ultimate particle sizes and the minimum spacings between slip bands*

Metal	Particle size (μ)	Minimum spacing between slip bands (μ)	
Al (room temperature)	2	2	(Brown, 1949)
Al (−180° C.)	~1*	0·5–1	(Brown, 1949)
Pb	6	4·2	(Andrade & Roscoe, 1937)
Cu	0·6	0·2–2	(Barrett, 1944)
Fe	~1	~ 2	(Paxton *et al.*, 1952)

* (Heidenreich, 1951)

Table 2. *Mean values of the angles between particles, dislocation densities, and the strength calculated after recovery*

Metal	Deformation (%)	α (°)	Dislocation density from measured angles between particles (cm.$^{-2}$)	Dislocation density from strength equation (cm.$^{-2}$)	Calculated strength after recovery (equation (4)) ($\times 10^8$ dyne cm.$^{-2}$)
Cu	~ 50	1·5	2×10^{10}	$8{\cdot}5\times10^{9}$	12
Fe	57	2	7×10^{9}	4×10^{9}	18
Al	57	3	9×10^{9}	$1{\cdot}3\times10^{9}$	40
Sn	35	2·5	$1{\cdot}5\times10^{9}$	$5{\cdot}6\times10^{8}$	13
Zn	15	1	8×10^{8}	$1{\cdot}7\times10^{8}$	8·3
Pb	12	0·5	4×10^{8}	$2{\cdot}3\times10^{8}$	4·9

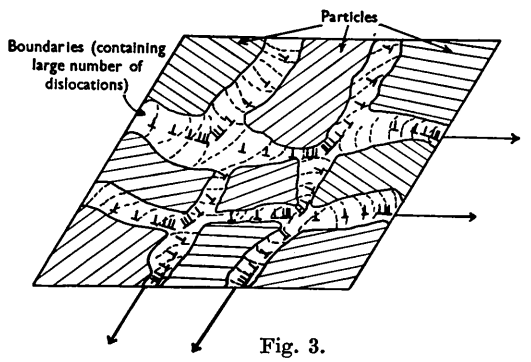

Fig. 3.

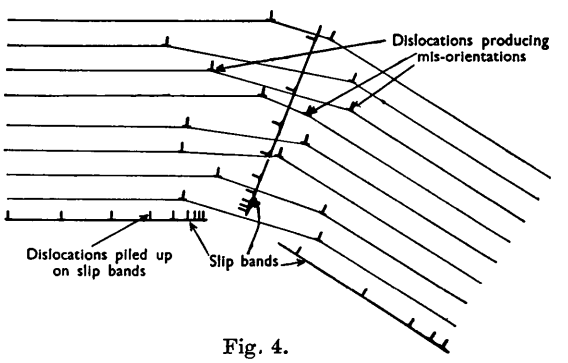

Fig. 4.

Fig. 3. Schematic diagram of a deformed grain. Arrows indicate mean direction of slip. Distance between arrows is average separation (= particle size).
Fig. 4. Schematic diagram of a possible distribution of dislocations in the boundaries.

rotation must occur at the slip bands. Heidenreich & Shockley (1948) and Wilman (1951) have reported the existence of rotations normal to the slip planes in single crystals. Honeycombe's (1951) X-ray micrographs from deformed aluminium single crystals show the existence not only of large misorientations across a kink band (as pointed out by that author) but also of other misorientations of the slip bands relative to each other (see Figs. 12, 13 of original paper).

(4) For small deformations, lamellae are expected to be formed; it is found that for small deformations the spots within the arcs on microbeam photographs are often in the form of long parallel streaks (see Hirsch & Kellar, 1952, Fig. 6(*a*)) which could be due to reflexions from a series of slightly misorientated bent lamellae. The striations of the Laue spots mentioned above can be interpreted in the same way. This parallelism is largely lost after heavier deformation, as expected (see Hirsch & Kellar, 1952, Fig. 6(*b*)). The photographs of the hexagonal metal zinc are generally anomalous in that for all deformations the rings consist of a large number of parallel streaks, which is expected for metals with only one slip plane.

(5) The occurrence of slip bands should be accompanied by misorientations and, therefore, by X-ray asterism, and conversely, if no X-ray asterism is observed, there should be no slip bands. In cadmium, extended carefully by 100%, slip bands, kink bands and asterism were not observed, slip occurring by pure shear on very closely spaced thin lamellae (Honeycombe, 1951). Similar results, for aluminium deformed by pure shear, have been reported by Röhm & Kochendörfer (1950).

Thus, although there is at present no direct evidence in support of the suggestion that the particles are the regions between slip bands, there are a number of experimental facts which can be explained simply on this hypothesis.

4. Distribution of dislocations

It is possible to derive from the X-ray photographs certain conclusions about the distribution of the dislocations producing the misorientations. An important feature of the photographs is the presence of background intensity between the spots along the Debye–Scherrer rings. This background is found only on the arcs of the rings which contain spots, and hence it must certainly arise from the boundary regions between the particles. In fact, the experiments show that *there is a continuous change of curvature across a boundary from one particle to the next, i.e. the particles cannot be produced by fragmentation* (a process which may give rise to discontinuities or gaps in the lattice). Further, the broadening of the diffuse background is no larger than that of the spots; this breadth corresponds to a maximum elastic curvature of a few minutes, whereas the angle across the boundaries is of the order of a few degrees (cf. Table 2). Thus it follows that *the boundary regions are plastically curved*, so that they contain large numbers of excess dislocations of one sign. On the average the total intensity of the scattering from one

boundary is of the same order of magnitude as that from a particle for the harder metals, whereas in the softer metals it is only a few per cent of that from a particle. This fact indicates that the dislocations producing the curvature are distributed over relatively large volumes in the hard metals. If the curvature is in the nature of a rotation normal to the slip plane, produced by screw dislocations, these cannot be arranged on a single slip plane as for a twist boundary (van der Merwe, 1950); if, on the other hand, the curvature is due to some dislocations inside the particles piled up against a barrier (fine slip; this suggestion is due to Prof. Mott), these cannot be arranged in a polygon wall. *The rotations are in the nature of either a gradual twist or a gradual bending at the slip band barriers.* At present there is some evidence in support of both these possibilities, but further experiments on the nature of the misorientations are required to determine the exact distribution of dislocations at the boundaries. Figs. 3, 4 show schematically the 'foam' structure and a possible distribution of dislocations in the boundaries in terms of edge dislocations only.

The density of excess dislocations of one sign can be determined from the mean angle between the particles. The method is described in detail elsewhere (Gay, Hirsch & Kelly, 1953); only the results are quoted here (Table 2). The estimates of dislocation density are much lower than the value of 10^{12} lines cm.$^{-2}$ which has usually been accepted. This latter figure is based mainly on the measurements of the energy stored in cold-worked metals (Taylor & Quinney, 1934). Recently Clarebrough, Hargreaves, Michell & West (1952) showed that the amount of energy stored depends on the purity of the metal. These authors found that the amount of energy stored in heavily cold-worked pure copper was much smaller than Taylor & Quinney's values; their experimental value would lead to a density of dislocations of $1{\cdot}4\times10^{11}$ cm.$^{-2}$. It is possible, therefore, that the low values of the dislocation density found in the microbeam experiments are due to the high purity of the metal used; on the other hand there may be a large number of dislocations present which do not contribute to the curvature, but no evidence is available from the X-ray photographs about such dislocations. This point will be discussed in detail in the next section.

5. The process of deformation and the strength of pure metals

We imagine that the process of deformation begins by the formation of slip bands on a single slip plane, and that the dislocations on these become locked at kink bands. Further deformation leads to slip on other planes and to a locking of the dislocation by the formation of sessile dislocations. The dislocations on the slip bands and any other dislocations (e.g. due to fine slip) pile up at the barriers. *After sufficiently heavy deformations a 'foam' structure is formed consisting of regions of relatively small dislocation density separated by regions of high dislocation density.*

If the dislocations piled up on the slip bands against the barriers are edge dislocations, these will not produce curvature; such a distribution of edge dislocations is likely to occur, and the total density of dislocations could therefore be greater than that estimated from the dislocations producing curvature (see above). It is possible to estimate the number of dislocations on a slip band from the calculations of Eshelby, Frank & Nabarro (1951) as follows.

The yield strength P of a region of metal containing n dislocations piled up on a slip band against a barrier is given by

$$P = nGs/\pi t\,, \qquad (1)$$

where G = rigidity modulus, s = Burgers vector, t = length of slip band containing the locked dislocations.

If we identify t with the average particle size, the value of n, and therefore the dislocation density, can be calculated, assuming theoretical values of yield stress discussed below (Table 3). The results shown in Table 2 are of the same order as, but generally lower than, the values obtained from the angles between the particles. It appears, therefore, that a distribution of dislocations on the slip bands which produces no curvature would not be expected to increase by an order of magnitude the estimated total density of dislocations.

Table 3. *Comparison of calculated and experimental values of strength of metals*

Metal	Calculated strength after deformation (equation (3)) ($\times10^8$ dyne cm.$^{-2}$)	Experimental strength ($\times10^8$ dyne cm.$^{-2}$)
Cu	17·5	23
Fe	20	32
Al	5·7	7
Sn	0·95	2·5
Zn	3·5	12–30
Pb	0·87	2·1

The 'foam' structure, which is suggested by the experiments, consists of soft regions of crystal separated by relatively hard regions. Plastic deformation can only occur if some yield takes place at the relatively hard boundary regions. Strain hardening is due to the formation of these hard regions (containing many dislocations) by the locking of the dislocations. A high stress concentration is necessary before the boundary is broken. According to Eshelby, Frank & Nabarro (1951), such a stress concentration can be produced by a row of dislocations on a slip band piled up at an obstacle. The stress τ due to such a row of dislocations is

$$\tau \sim P\sqrt{(t/x)}\,, \qquad (2)$$

where P is the applied stress and x is the distance ahead of the obstacle for which the stress is calculated.

Frank (1950) argues further that yield will occur when a new dislocation loop is formed ahead of the obstacle; the new loop must be of a critical length to ensure expansion. Thus the value of x is the sum of two terms, one of which represents the thickness of the obstacle and the other the critical length of the loop created at a particular stress. Energy considerations suggest that only dislocation loops whose critical radii are as small as $5s$ (where s is the Burgers vector) can be formed at a finite rate at room temperature. The critical stress for such loops is $\tau = G/25$, where G is the rigidity modulus. Reasonable limits of the thickness of the obstacle may be set at $5s$ and $50s$. These two values, together with the estimate for τ, lead to two values of P, equal to $(G/8)\sqrt{(s/t)}$ and $(G/3)\sqrt{(s/t)}$. As an average we may choose the value

$$P = (G/5)\sqrt{(s/t)}\,, \qquad (3)$$

a formula given by Mott (1952), which corresponds to a thickness of boundary of $20s$. Calculations of the final yield stress have been made from this relationship, identifying t with the particle size determined in the experiments. Good agreement is found between the calculated and experimental values, except for zinc, which is the only metal with one slip system (Table 3). The experimental values are taken from standard tables, except for aluminium and iron for which the strengths were measured on specimens of purity comparable to that of the specimens used for the X-ray work.

In this mechanism slip is transferred from particle to particle by the creation of a new dislocation source, whilst the leading dislocation of the stopped row causes some change of shape at the boundary. The strain hardening of a metal is to be associated in the first place with a decrease in t, i.e. an increase in the number of locks per unit length of slip band, due to kink bands and glide on a different slip system. After a constant particle size has been reached and the foam structure is completed, the stress–strain curve is found to be comparatively flat (Kelly, 1953). Nevertheless some further strain-hardening occurs and this is accompanied by an increase in the mean angle between particles; this implies a small increase in the number of excess dislocations in the boundaries, i.e. an increase in the angles between the particles causes a hardening of the boundaries. It may be noted that according to this model only a relatively small proportion of the dislocations is responsible for the propagation of slip (namely those piled up on the slip bands), and that these dislocations probably do not produce curvature.

6. Recovery

The changes in the physical properties of metals with time after deformation are accompanied by certain structural changes. For aluminium, the diffraction spots sharpen, i.e. the distortion of the particles is reduced; further, the background between spots disappears and is replaced by reflexions from a number of small particles, thereby reducing the average particle size slightly (Hirsch, 1952*b*). For the harder metal, copper, no significant changes could be detected at room temperature in the X-ray pattern; iron and nickel have not yet been examined. For the softer metals (Cd, Sn, Pb and Zn), the reduction of the distortion takes place more rapidly; the heavily distorted material disappears, but no increase in the number of particles for these metals has yet been observed. In some specimens, new grains are observed which grow with time after deformation. Although this will also cause a decrease in the yield stress, the process is more correctly called recrystallization. *True recovery is due to a removal of the strains and distortion within the particles, and it is not accompanied by any growth.* These results are in agreement with those of other workers (Megaw & Stokes, 1945; Heidenreich, 1951). As expected, recovery takes place more rapidly for the softer metals; the rate of recovery increases with temperature.

It is found that the angles between particles do not change perceptibly during recovery *and hence the number of excess dislocations in the boundaries remains approximately constant.* Thus recovery is due to a rearrangement of dislocations in the boundaries, and the possible diffusion of dislocations in the particles to the boundaries. If there are only dislocations of one sign in the boundaries, then probably simple polygonization takes place (Cahn, 1949); but if the dislocations of either sign are present, these must annihilate each other, leaving only the excess dislocations of one sign which polygonize. Recovery, therefore, may be due to polygonization and a possible mutual annihilation of equal numbers of dislocations of either sign occurring in the boundaries. The results indicate that generally only one wall is formed per boundary, except in the case of aluminium where sometimes more than one wall per boundary is formed. Many of the dislocations on the slip bands must also polygonize, since after recovery the distortion of the particles is small.

It is interesting to calculate the yield stress after recovery. If the dislocations form single walls after recovery, the strength might be determined by the stress required to pull a dislocation away from the wall. Burgers (1939) has calculated the stress on a 'pulled out' dislocation; its maximum value is

$$\sim \frac{G}{2\pi(1-\nu)} \cdot \frac{s}{h}\,, \qquad (4)$$

where h is the distance between neighbouring dislocations in the walls, and ν is Poisson's ratio. In the microbeam experiments, it is possible to determine the angle $\alpha = s/h$ between particles. Some values of α are given in Table 2. Using these values, the strengths may be computed according to the equation above (Table 2). The values obtained for some metals are in excess of the strength before recovery (cf.

Table 3); this model therefore is not satisfactory for these metals and it is probable that the strength depends once again on the stress concentration necessary to overcome a rather softer obstacle.

7. Recrystallization

Spontaneous recrystallization may take place during or immediately after the deformation of the softer metals. In these metals, a limiting value of the particle size is reached after only small deformations; continued deformation does not significantly change the size of the particles before complete recrystallization occurs. Although the particle size is unchanged, the angle between the particles is increased by further deformation (see Fig. 1). Thus the onset of recrystallization must be associated with a maximum angle between adjacent particles; this implies that the *metal recrystallizes when the density of excess dislocations in the boundaries exceeds a limiting value.* Growth will probably start in the region of maximum dislocation density. This is in agreement with the observations of other investigators, e.g. Jacquet (1945) found that nuclei in copper crystals appear on slip bands, Collins & Matthewson (1940) found that new grains often have boundaries parallel to deformation bands, and Crussard & Aubertin (1946) found that nuclei are formed at curved twin boundaries in zinc.

The presence of a few recrystallized grains at small deformations is probably due to a particularly high local dislocation density; this might well occur at kink bands during the early stages of the deformation. Complete recrystallization will take place when the dislocation density exceeds the limiting value at many boundaries.

Recrystallization may take place in any of the following ways: (1) by a process of random nucleation in the regions of maximum dislocation density, (2) by growth of the particles formed during the deformation, (3) by production of a nucleus at a boundary, with orientation similar to that of neighbouring particles (e.g. by polygonization of the boundary; cf. Cahn, 1950).

For all the metals examined (except tin, see below) no direct evidence for any of these processes has been found, but some indirect evidence is available. When complete spontaneous recrystallization occurs the recrystallized grains are randomly orientated. If the recrystallization were due to growth of the particles formed during the deformation an arc structure might be expected, particularly in those cases in which the recrystallized grain size is of the same order as the final particle size after deformation (i.e. there is roughly one nucleus per boundary).

In those cases where recrystallization occurs after annealing, any of the processes is possible, but as recovery precedes recrystallization, the second mechanism is more likely to occur. But only for tin has direct evidence been found for the growth of particles. Actually in tin several particles of the same grains were found to grow simultaneously, while some recrystallized grains (revealed by Laue spots) disappeared. It is probable that the original grain grows as a whole; such a process is similar to that of strain-induced boundary migration reported by Beck & Sperry (1950).

Thus the experiments indicate that recrystallization takes place when the dislocation density exceeds a certain value, and that both random nucleation and growth of particles can occur.

PART III

8. Line broadening and the strength of metals

The experimental methods which have been used in this work are in many ways complementary to the line-broadening experiments carried out by other authors; it is interesting to compare the results obtained by the different techniques. On the present picture, the cold-worked metals contain particles separated by severely distorted boundary regions; the particles themselves are very often distorted, but the distortion is less severe than in the adjacent boundaries.

The results obtained by the X-ray techniques employed in this research are generally in agreement with those obtained by Stokes, Pascoe & Lipson (1943), Smith & Stickley (1943), Paterson (1948), Hall & Williamson (1951*a*, *b*), Warren & Averbach (1952) and others. All these workers have found that strain broadening predominates, whilst the particle-size broadening is relatively small. The microbeam experiments show that the particle size in most cold-worked metals is of the order of a micron, but that these particles are strained. The distortion of the particles is of the same order as that found by line-broadening experiments (cf. Hirsch, 1952*b*; Gay & Kelly, 1953*a*). The values of particle size determined from line-broadening measurements are, however, not always in agreement with the values obtained using the microbeam technique (see Table 4); in particular, the values obtained by Wood and his colleagues

Table 4. *Comparison of limiting particle size determined by microbeam and line-broadening methods*

Metal	Particle size by microbeam method ($\times 10^{-4}$ cm.)	Particle size by line-broadening method ($\times 10^{-4}$ cm.)
Cu	0·6	0·6 (Dehlinger & Kochendörfer, 1939) 0·07 (Wood, 1939) ~0·1 (Hall & Williamson, 1951*b*)
Fe	~1	>1 (Auld & Garrod, 1952) 0·03 (Wood & Rachinger, 1949*a*)
Al	2	~1 (Hall & Williamson, 1951*b*) ~1 (Wood, 1939)

Table 5. *Calculations of strength using the Bragg criterion, and of stresses within the particles from the experimental broadenings*

Metal	Strength using Bragg criterion ($\times 10^8$ dyne cm.$^{-2}$)	Experimental broadening (radians)	Stresses calculated from experimental broadening ($\times 10^8$ dyne cm.$^{-2}$)	Experimental strength ($\times 10^8$ dyne cm.$^{-2}$)
Cu	1·78	$2{\cdot}3\times10^{-3}$	29·6	23
Fe	1·14	$15{\cdot}3\times10^{-3}$	96·5	32
Al	0·34	$1{\cdot}3\times10^{-3}$	9·1	7
Sn	0·013	—	—	2·5
Zn	0·11	—	—	12–30
Pb	0·031	—	—	2·1

agree with those obtained by the present authors only for aluminium.

The interpretation of the earlier results of Wood (1939) is rather doubtful because of the effect of strain broadening; however, in more recent experiments (Wood & Rachinger, 1949*a*) it has been claimed that this effect has been eliminated. The wavelength test which has been described appears to confirm this, but recently some doubt has been thrown on this interpretation, and results have been obtained which indicate that most of the broadening is due to strains (Auld & Garrod, 1952).

Wood & Rachinger (1949*a*) have made estimates of the final strengths of metals by means of the Bragg criterion (Bragg, 1949) i.e. $T = Gs/t$, where T is the tensile strength, G the rigidity modulus, s the slip distance, and t the particle size. Now this criterion is merely an expression of the fact that the energy of a strained block is the same before and after slip. The actual process of slip cannot take place until a pair of dislocations is created within the particle, and this requires a very much larger stress. Thus the Bragg criterion must be regarded as giving only a lower limit of T. Values of T calculated according to this criterion are shown in Table 5, and it can be seen that this condition is satisfied. It must be doubtful, therefore, if the use of the Bragg criterion by Wood & Rachinger is of any significance.

It should also be pointed out that the argument used by Wood & Rachinger is inconsistent with the interpretation of the X-ray photographs presented in their paper. It is argued that, on the average, every block is strained by an amount $\frac{1}{2}Gs/t$ before slip occurs. On this basis, linear strains $\sim\pm\frac{1}{4}s/t$ are expected in the metal, some of the particles having slipped, whilst others are on the point of slip. It follows that there must be strain broadening $\beta_\eta \sim \frac{1}{2}(s/t)\tan\theta$. If β_t is the particle-size broadening,

$$\frac{\beta_\eta}{\beta_t} = \frac{\frac{1}{2}(s/t)\tan\theta}{\lambda/t\cos\theta} = \frac{s\sin\theta}{2\lambda} = \frac{s}{4d}\,.$$

But the conditions of the experiments are such that β_η is always of the order of β_t, and thus strain broadening cannot be neglected if the interpretation put forward by Wood & Rachinger is to be consistent.

The apparent agreement between calculated and observed values of T claimed by Wood & Rachinger may, in fact, have the following explanation. Let it be assumed that the broadenings observed in the experiments are due to strains and not to the effect of particle size. Then the average strain across a particle is

$$\eta = \tfrac{1}{2}\beta/\tan\theta = \lambda/2t\cos\theta\tan\theta = d/t\,,$$

where t is the apparent particle size deduced by Wood. Now the shear stress across the particle is

$$\sim 2G\eta = G\frac{2d}{t} = \tfrac{1}{2}G\frac{s}{t}\cdot\frac{4d}{s}\,.$$

As before, $4d/s \sim 1$, so that the average shear stress is $\sim \frac{1}{2}Gs/t$. Thus the agreement obtained by Wood & Rachinger is an indication that the broadenings are predominantly due to strains. This interpretation is in agreement with the present results. Table 5 shows the magnitude of the distortion broadenings from the particles; if these are interpreted as being caused by elastic strains (due to bending), the stresses so obtained are of the order of the yield strength of the material.

In the description of some recent work on the creep of polycrystalline aluminium, Wood & Rachinger (1949*b*) again suggest that the Bragg criterion may be used to give the strength of the metal. In these experiments estimates of the particle size within the metal were obtained from the number of spots on the diffraction rings. However, the appearance of the X-ray photographs suggests that the values of particle size deduced by these authors are too small. Although no details of the determination of particle size are given it would appear that the discrepancy can be explained if the authors have assumed that the number of spots on a particular diffraction ring is inversely proportional to the linear particle size. However, unless the size of the particles is large compared with the penetration of the X-ray beam, the number of spots on a particular diffraction ring is inversely proportional to the *volume* of the particles (Hirsch & Kellar, 1952). If

$$N = \frac{\text{number of spots on a particular ring on photographs of the deformed material}}{\text{number of spots on the same ring on photographs of the original material}}\,,$$

then $t = t_0/N^{\frac{1}{3}}$ (t, t_0 are the deformed and original particle sizes respectively), not $t = t_0/N$, as has ap-

parently been used by Wood & Rachinger. If it is assumed that the original particle size is $\sim 100\mu$, the 'cell' size at 350° C. is $\sim 40\mu$ (instead of 7μ as published by Wood & Rachinger), or at 300° C. $\sim 35\mu$ (instead of 5μ). The estimate of 40μ is of the same order as the 'cell' dimensions determined from the micrograph in the original publication, and so explains the apparent discrepancy between the particle size determined using X-ray methods and the micrographic cell size which has already been noted by Guinier (1952). Further, the particle size at 150° C. is $\sim 10\mu$, and at 20° C. $\sim 4\mu$; these estimates are made from the appearance of the photographs by comparison with those taken by the authors under similar circumstances. Wood & Rachinger claim agreement between the experimental yield stress and values calculated from the expression Es/t, where E is Young's modulus. If the correct values of t are substituted the calculated values of stress will be lower than the actual yield stress by factors of 4 and 7 respectively. Values of strength estimated from the original Bragg criterion, i.e. Gs/t will be low by factors of 10–20; this is in agreement with the present results (see Hirsch, 1952*b*; also Table 5). Thus it may be concluded that the present results are in good agreement with those of other workers.

9. Conclusions

The interpretation of the experimental results which has been put forward in this paper may not be unique or comprehensive; it is, however, supported by many of the experimental data. The main features of the model may be summarized as follows:

(*a*) The grains within a cold-worked metal contain a 'foam' structure of relatively perfect particles linked continuously with each other by distorted boundary regions. The formation of a structure of this kind is a direct result of the accumulation of dislocations on slip bands during cold work; the structure is not formed by a recovery process subsequent to cold working. This model differs from previous particle models of the cold-worked state, for the particles are not produced by fragmentation and the boundaries between the particles cannot be regarded either as normal intercrystalline boundaries or polygon boundaries. *In the present model, particles are considered to be regions of relatively small plastic curvature, whilst the boundaries are regions of large plastic curvature.*

(*b*) The boundaries of the particles are the slip bands; the particles are the regions between the slip bands.

(*c*) The exact distribution of dislocations at the boundaries is uncertain at present, but the twists or bends are distributed over volumes of material large compared with those associated with a cross-grid of screw dislocations (Frank, 1948; van der Merwe, 1950) or with a polygon wall.

(*d*) The strength of metals is determined by the locking of dislocations on the slip bands. The initial strain hardening is due to an increase in the number of locks per unit length of slip band; the final hardening is due to a hardening of the boundaries. The interpretation of Wood's results in terms of the Bragg criterion is unsatisfactory.

(*e*) Estimates of the excess dislocations in the boundaries are smaller than the usually accepted value of 10^{12} lines cm.$^{-2}$. In order to assess the significance of this difference, the energy stored during cold work in the specimens used in the present experiments should be determined, since the experimental measurements of energy (on which the value of 10^{12} lines cm.$^{-2}$ is based) are critically dependent on the purity of the metal.

(*f*) Recovery is due to a rearrangement of the dislocations in the boundaries to reduce the strain energy. The detailed process is uncertain, although polygonization may play an important part. The number of excess dislocations of one sign in the boundaries remains unchanged during recovery.

(*g*) Recrystallization occurs where the number of excess dislocations of one sign exceeds a limiting value.

An important feature of these experiments has been the definition of further experimental research to be carried out. Specific experiments which must be undertaken may be listed: (*a*) the determination of the relationship between the particles and the metallographically observed slip bands, deformation bands etc., (*b*) the determination of orientation relationships to enable the distribution of dislocations to be established, and (*c*) the elucidation of the origin of the orientation differences. It is hoped that further work, now in progress, will throw light on these problems.

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The Crystal Structure of Rh_2B*

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The crystal structure of Rh_2B has been determined from X-ray rotation and Weissenberg photographs. The unit cell is orthorhombic, with dimensions $a = 5{\cdot}42 \pm 0{\cdot}01$, $b = 3{\cdot}98 \pm 0{\cdot}02$, $c = 7{\cdot}44 \pm 0{\cdot}03$ Å. The space group is D_{2h}^{16}–*Pnma* with eight rhodium atoms in the unit cell. The atomic coordinates were found by means of Booth's method of 'steepest descents', using the intensity data from the *hk*0 and 0*kl* zones. The relation of this structure to other intermetallic borides is discussed and some comments are made on the type of intermetallic bonding which may exist.

Introduction

Boron is well known for its ability to form binary compounds with many metals. The crystal structures of many of these borides have been studied by Kiessling, and a survey of some of his results has recently been published (Kiessling, 1950). In addition, a survey of the intermetallic compounds formed between boron and the noble metals was recently made (Buddery & Welch, 1951) in which it was shown that these compounds are easily prepared by direct synthesis and

* The opinions contained herein are those of the authors and are not to be construed as official or as reflecting the views of the Navy Department or the Naval Establishment at large.

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